Neutrino Theory — Overview

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(image credits: Wikipedia, E. Palti)
A wealth of discoveries in neutrino physics since 1998...

Some highlights:

1998: atmospheric $\nu_\mu$ disappearance (SK)
2002: solar $\nu_\epsilon$ disappearance (SK)
2002: solar $\nu_\epsilon$ appear as $\nu_\mu, \nu_\tau$ (SNO)
2004: reactor $\bar{\nu}_\epsilon$ oscillations (KamLAND)
2004: accelerator $\nu_\mu$ disappearance (K2K)
2006: accelerator $\nu_\mu$ disappearance (MINOS)

2011: accelerator $\nu_\mu$ appear as $\nu_\epsilon$ (T2K, MINOS)
2012: reactor $\bar{\nu}_\epsilon$ disappear (Daya Bay, RENO)

2012: reactor angle measured!

2014: hint for CP violation? (T2K)
2015: hints for normal hierarchy? (SK, T2K, NOvA)
2016: hint for non-maximal atm mixing? (NOvA)

Signals physics beyond the Standard Model (SM)!
The emergent picture…

a (seemingly) robust 3-neutrino mixing scheme

\[ U_{\text{MNSP}} = R_1(\theta_{23}) R_2(\theta_{13}, \delta_{\text{MNSP}}) R_3(\theta_{12}) P \]

<table>
<thead>
<tr>
<th></th>
<th>NuFIT 3.0</th>
<th>Capozzi et al.</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{12}[^\circ] )</td>
<td>33.56\pm0.77</td>
<td>33.02\pm1.06</td>
<td>33.2\pm1.2</td>
</tr>
<tr>
<td>( \theta_{13}[^\circ] )</td>
<td>8.46\pm0.15</td>
<td>8.43\pm0.14</td>
<td>8.45\pm0.15</td>
</tr>
<tr>
<td>( \theta_{23}[^\circ] )</td>
<td>41.6\pm1.5</td>
<td>40.5\pm1.4</td>
<td>41.4\pm1.6</td>
</tr>
<tr>
<td>( \delta[^\circ] )</td>
<td>-99^\circ\pm59</td>
<td>-108^\circ\pm38</td>
<td>7.45\pm0.25</td>
</tr>
<tr>
<td>( \Delta m_{21}^2 \times 10^{-5} \text{eV}^2 )</td>
<td>7.50\pm0.19</td>
<td>7.37\pm0.17</td>
<td>2.55\pm0.05</td>
</tr>
<tr>
<td>( \Delta m_{31}^2 \times 10^{-3} \text{eV}^2 )</td>
<td>2.524\pm0.039</td>
<td>2.56\pm0.05</td>
<td>2.55\pm0.05</td>
</tr>
</tbody>
</table>

Global Fits:

Gonzalez-Garcia et al.,’14,’16
(www.nu-fit.org)
Capozzi et al.,’13,’16
Forero et al., ‘14

(image credits: King, Luhn)
Caveat: some anomalies in the data

1995: $\bar{\nu}_e$ appearance (LSND)
2007: $\bar{\nu}_e$ appearance (MiniBooNE)
2012: $\nu_e$ appearance (MiniBooNE)
1995: $\nu_e$ disappearance (Gallium)
2011: $\nu_e$ disappearance (Reactor)

$\sim$eV-scale sterile neutrino(s)?

But the situation is unclear:

Here set aside this possibility, focus on 3 active families only...
New questions, excitement for BSM physics!

Implications for the SM flavor puzzle:

what is the origin of the quark and lepton masses and mixings?

Goal: a satisfactory and credible theory of flavor (very difficult!)

Many questions:

- Majorana or Dirac neutrinos?
- Nature of neutrino mass suppression?
- Mass hierarchy?
- Lepton mixing angle pattern?
- CP violation?
- Implications for BSM paradigms?
- Connections to other new physics (NP)?
Mass Generation

**Quarks, Charged Leptons**

“natural” mass scale tied to electroweak scale

Dirac mass terms, parametrized by Yukawa couplings

\[ Y_{ij} H \cdot \bar{\psi}_Li \psi_{Rj} \rightarrow M_u, M_d, M_e \]

top quark: O(1) Yukawa coupling

rest: suppression (flavor symmetry)

**Neutrinos**

Main question: origin of neutrino mass suppression

**Options:** Dirac \[ \Delta L = 0 \]

Majorana \[ \Delta L = 2 \]
Majorana first: \( \Delta L = 2 \)

Advantages: naturalness, leptogenesis, \( 0\nu\beta\beta \)

SM at NR level: Weinberg dimension 5 operator

\[
\frac{\lambda_{ij}}{\Lambda} L_i H L_j H
\]

if \( \lambda \sim O(1) \) \( \Lambda \gg m \sim O(100 \text{ GeV}) \) (but wide range possible)

Underlying mechanism: examples

Type I seesaw \( \nu_R \) (fermion singlet)

Type II seesaw \( \Delta \) (scalar triplet)

Type III seesaw \( \Sigma \) (fermion triplet)

+ variations

(image credit: Dinh et al.)
Prototype: Type I seesaw

Type I: Minkowski; Yanagida; Gell-Mann, Ramond, Slansky; Mohapatra, Senjanovic; ...

right-handed neutrinos:

\[ Y_{ij} L_i \nu_{Rj} H + M_{Rij} \nu_{Ri} \nu^c_{Rj} \]

\[ \mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \]

\[ m \sim \mathcal{O}(100 \text{ GeV}) \quad M \gg m \]

\[ m_1 \sim \frac{m^2}{M} \quad m_2 \sim M \gg m_1 \]

\[ \nu_{1,2} \sim \nu_{L,R} + \frac{m}{M} \nu_{R,L} \]

advantages: naturalness, connection to grand unification, leptogenesis, ...

disadvantage: testability (even at low scales)

Different in Type II, III: new EW charged states — visible at LHC?

Type II: Konetchsy, Kummer; Cheng, Li; Lazarides, Shafi, Wetterich; Schecter, Valle; Mohapatra et al.; Ma; ...

Type III: Foot, He, Joshi; Ma; ...
Radiative neutrino mass generation:

can have other NR operators in SM with \( \Delta L = 2 \) (odd mass dimension \( d > 5 \))

\[
\begin{align*}
\text{d=7:} & \quad LLL\ell^c H \\
& \quad LLQd^c H \\
& \quad LLQ\bar{u}^c H \\
& \quad L\bar{e}^c \bar{u}^c d^c H \\
\text{d=9:} & \quad LLL\ell^c \ell^c \quad \text{(Zee, Babu)} \\
& \quad LLQd^c Qd^c \\
& \quad \text{and many others…}
\end{align*}
\]

“open up” operator with SM-charged states, close external legs: loops

NP scale can be accessible at LHC! of course, subject to LFV bounds

One way in which leptoquarks can appear…

e.g. scalar leptoquark \( \phi \sim (3, 1, -1/3) \)

+ octet fermion \( f \sim (8, 1, 0) \)

Potential relevance for B-physics anomalies…

(example of Bauer-Neubert leptoquark ’15)

Cai, Gargalones, Schmidt, Volkas ’17
Many other ideas for Majorana neutrino masses...

more seesaws (double, inverse,...),
SUSY with R-parity violation, RS models...

lepton number violation → Majorana $\nu$ masses

Now for Dirac neutrino masses:

Require strong suppression $Y_{\nu} \sim 10^{-14}$

Less intuitive, but mechanisms exist...

extra dimensions, new gauge symmetries (non-singlet $\nu_R$),
SUSY breaking effects, string instanton effects,...

General themes:

Trade-off b/w naturalness and testability.
Much richer than quark and charged lepton sectors.
Lepton mixings

\[ \mathcal{U}_{\text{MNSP}} = \mathcal{R}_1(\theta_{23}) \mathcal{R}_2(\theta_{13}, \delta) \mathcal{R}_3(\theta_{12}) \mathcal{P} \]

Pontecorvo; Maki, Nakagawa, Sakata

\begin{align*}
(\mathcal{U}_{\text{MNSP}})_{ij} = & 
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_{23} & \sin \theta_{23} \\
0 & -\sin \theta_{23} & \cos \theta_{23}
\end{pmatrix} \\
& \begin{pmatrix}
\cos \theta_{13} & 0 & \sin \theta_{13} e^{i\delta} \\
0 & 1 & 0 \\
-\sin \theta_{13} e^{-i\delta} & 0 & \cos \theta_{13}
\end{pmatrix} \\
& \begin{pmatrix}
\cos \theta_{12} & \sin \theta_{12} & 0 \\
-\sin \theta_{12} & \cos \theta_{12} & 0
\end{pmatrix}
\end{align*}

diagonal phase matrix (Majorana neutrinos)

Reactors angle last measured but now best known!

\[ \sin^2 2\theta_{13} = 0.0841 \pm 0.0027 \text{(stat.)} \pm 0.0019 \text{(syst.)} \]

Daya Bay (most recent result)

“Dirac” phase \( \delta \) unconstrained at \( 3\sigma \) but best fit consistent with \( \delta \sim -\frac{\pi}{2} \) (also T2K hint)

Compare quarks:

\[ \mathcal{U}_{\text{CKM}} = \mathcal{R}_1(\theta_{23}^{\text{CKM}}) \mathcal{R}_2(\theta_{13}^{\text{CKM}}, \delta^{\text{CKM}}) \mathcal{R}_3(\theta_{12}^{\text{CKM}}) \]

Cabibbo; Kobayashi, Maskawa

\begin{align*}
\theta_{12}^{\text{CKM}} &= 13.0^\circ \pm 0.1^\circ = \theta_C \quad \text{(Cabibbo angle)} \\
\theta_{23}^{\text{CKM}} &= 2.4^\circ \pm 0.1^\circ \\
\theta_{13}^{\text{CKM}} &= 0.2^\circ \pm 0.1^\circ \\
\delta^{\text{CKM}} &= 60^\circ \pm 14^\circ
\end{align*}

3 “small” angles, 1 “large” phase
Lepton mixings

Certainly **two large mixing angles**: $\theta_{23}, \theta_{12}$

**Dirac phase**: too soon to say, but intriguing hints)

**Majorana phases**: unlikely to know anytime soon**

A main question: is $\theta_{13}$ large or small?

Large reactor angle: **vs.** Small reactor angle:

the case for **anarchy**

the case for structure (symmetry)

2012 reactor angle meas. near prior upper bound:

renewed interest in flavor anarchy

de Gouvea and Muryama ’12
Altarelli et al. ’12, Bai and Torroba ’12,...

(also popular approach for NP flavor violation at scales $\sim$10 TeV)

Baumgart et al. ’15,...
Family symmetries (structure)

Very different structure for leptons and quarks!

Quarks:

spontaneously broken family symmetry at scale $M$

$$Y_{ij} H \cdot \bar{\psi}_L \psi_R \rightarrow \left( \frac{\varphi}{M} \right)^{n_{ij}} H \cdot \bar{\psi}_L \psi_R$$

small mixings and hierarchical masses:

continuous family symmetry

both Abelian and non-Abelian: many examples!

$\mathcal{M}_u, \mathcal{M}_d$ approx diagonalized by same unitary transformation

(can can choose basis where both close to diagonal)

$\mathcal{U}_{\text{CKM}} = \mathcal{U}_u \mathcal{U}_d^\dagger \sim 1 + \mathcal{O}(\lambda)$

$\lambda \sim \frac{\varphi}{M}$

Wolfenstein parametrization: $\lambda \equiv \sin \theta_c = 0.22$

suggests Cabibbo angle (or some power) as a flavor expansion parameter
Leptons:

For the **charged leptons**: hierarchical masses → similar strategy?

But now, in basis where $M_e$ is diagonal, $M_\nu$ is not diagonal:

$M_\nu$ diagonalization requires 1 small, 2 large mixing angles!

Arguably the **most challenging*** pattern: (* for three families)

\[
\begin{align*}
\text{small angles} & \quad \longrightarrow \quad \sim \text{diagonal } M_\nu \\
\text{large, small} & \quad \longrightarrow \quad \sim \text{Rank} M_\nu < 3 \\
\text{large angles} & \quad \longrightarrow \quad \text{anarchical } M_\nu \\
\text{small, large} & \quad \longrightarrow \quad \text{fine-tuning, non-Abelian}
\end{align*}
\]

relatively straightforward at leading order

→ A model-building opportunity!
Lepton mixings:

No unique theoretical starting point for the flavor expansion!

\[ \mathcal{U}_{\text{MNSP}} \sim \mathcal{W} + O(\lambda') \]

mixing angles \((\theta^\nu_{12}, \theta^\nu_{23}, \theta^\nu_{13})\) (diagonal charged lepton basis)

“Bare” mixing angles generically shift due to \(O(\lambda')\) corrections

\[ \theta^\nu_{13} = 0 \quad \theta_{13} \sim \frac{\lambda_C}{\sqrt{2}} \]

A priori, expansions in quark and lepton sectors unrelated.

Unification paradigm (broad sense): set \(\lambda' = \lambda_C\)

ideas of quark-lepton complementarity and “Cabibbo haze”

Raidal ’04, Minakata+Smirnov ’04, many others...
(“haze” terminology from Datta, L.E., Ramond ’05)

Pre-measurement, speculation that reactor angle is a Cabibbo effect

Ramond ’04,...
Possible starting points:

Most studied: maximal atmospheric, zero reactor

\[ \theta_{23}^\nu = \frac{\pi}{4} \quad \theta_{13}^\nu = 0 \]

classify scenarios by bare solar angle

- **tri-bimaximal mixing**: \( \sin^2 \theta_{12}^\nu = 1/3 \)
  - [Harrison, Perkins, Scott '02; Xing '02; He, Zee '02; Ma '03...]

- **bimaximal mixing**: \( \sin^2 \theta_{12}^\nu = 1/2 \)
  - [Vissiani '97; Barger et al. '98; Baltz, A. Goldhaber, M. Goldhaber '98;...]

- **golden ratio (A) mixing**: \( \sin^2 \theta_{12}^\nu = 1/(2 + r) \sim 0.276 \)
  - [Datta, Ling, Ramond '03; Kajiyama, Raidal, Strumia '08;...]
  - \( r = (1 + \sqrt{5})/2 \)

- **golden ratio (B) mixing**: \( \sin^2 \theta_{12}^\nu = (3 - r)/4 \sim 0.345 \)
  - [Rodejohann '09;...]

- **hexagonal mixing**: \( \sin^2 \theta_{12}^\nu = 1/4 \)
  - [Albright, Duecht, Rodejohann '10, Kim and Seo '11;...]

Also can study scenarios without \( \theta_{13}^\nu = 0 \)

- [Lam '13; Holthausen et al. '12; Hagendorn... many others...]

All can be obtained via discrete non-Abelian family symmetries
Model-building approach

Choose a discrete non-Abelian group for family symmetry

Options: $SU(3)$, $SO(3)$ subgroups:

$A_4$, $S_4$, $A_5$, $\Delta(3n^2)$, $\Delta(6n^2)$, $D_n$, $T'$, $I'$, ...

Example (Majorana $\nu$):

Flavons:

$\phi^l$, $\phi^\nu$

Residual symmetries:

$T\langle \phi^l \rangle \approx \langle \phi^l \rangle$

$S, U \langle \phi^\nu \rangle \approx \langle \phi^\nu \rangle$

(or broken further, e.g. only $S$ or $U$ unbroken)

Corrections in flavor expansion: (i) NLO in flavons, (ii) “charged lepton”/kinetic/RG...

Many papers! Some authors (not comprehensive): King, Ma, Ding, Feruglio, Lam, Rodejohann, Chen, Hagedorn, Luhn, Stuart, LE...
Example: tri-bimaximal mixing (TBM/HPS)

$\mathcal{U}^{(\text{HPS})}_{\text{MNSP}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

($\sim$Clebsch-Gordan coeffs!)

Meshkov, Zee…

Many models pre-dated reactor angle measurements clearly current data requires Cabibbo-sized corrections

see e.g. Albright et al. ’10

Prototypical scenarios: $A_4$ $S_4$ $T'$ (typically SUSY/SUSY-GUT)

Many, many authors!!

Ma et al.; Altarelli, Feruglio; Carone et al.; Chen et al.; King et al.; Ding; Lam…

“minimal” flavor group (contains $S, T, U$ generators)

Lam; Ding et al;…

Residual symmetries: $\mathbb{Z}_3 \sim T \quad \mathbb{Z}_2 \times \mathbb{Z}_2 \sim S, U, SU$ (Klein symmetry)

Can further break down Klein symmetry:

1 column only of HPS matrix preserved: TM1, TM2 + corrections

see King ’17 for review
CP Violation

Consider case of spontaneous CP violation — calculable phases.

Idea of generalized CP: \( X^T M_\nu X = M_\nu^* \), \( Y^\dagger M_e M_e^\dagger Y = (M_e M_e^\dagger)^* \)

“ordinary” CP has \( X = Y = 1 \)

Grimus, Rebelo '95

automorphisms of discrete family symmetry:

\[
X \rho(g) X^{-1} = \rho(g')
\]

(consistency condition)

family symmetry

Residual/generalized CP symmetries

existence of “CP basis”

group classification:

Holthausen, Lindner, Schmidt '12
Chen et al. '14

Lots of interesting recent work along these lines!

many recent papers! see King '17 for review
Residual/CP symmetries (model-independent approach)

Assumptions: Majorana neutrinos, full Klein symmetry preserved

\[ U_\nu^T M_\nu U_\nu = M_\nu^{\text{diag}} \quad \text{invariant if} \quad U_\nu \rightarrow U_\nu Q_\nu \quad Q_\nu = \text{Diag}(\pm 1, \pm 1, \pm 1) \]
\[ \text{Det} Q_\nu = 1 \]

From these, obtain diagonal Klein generators

\[ (G_i^{\text{diag}})^T M_\nu^{\text{diag}} G_i^{\text{diag}} = M_\nu^{\text{diag}} \]

Then obtain Klein generators:

\[ G_i = U_\nu G_i^{\text{diag}} U_\nu^\dagger \quad G_i^T M_\nu G_i = M_\nu \]

(reconstruct from MNSP for diagonal charged leptons)

For generalized CP operators in neutrino sector:

from above and

\[ X_\nu G_i^* - G_i X_\nu = 0 \]

\[ X_i X_i^* = G_0 \quad X_0 X_i^* = G_i \quad X_i X_j^* = G_k \]

Similar approach for charged lepton generalized CP:

but need to be careful of phase redefinition degrees of freedom
SUSY GUTs and String Models

SUSY GUTs: explicit realizations of these scenarios (+ quark sector)

recent example: \[
\text{SUSY Pati-Salam} \quad \text{Poh, Raby, Wang '17}
\]

\[
SU(4)_C \times SU(2)_L \times SU(2)_R \quad D_3 \times U(1) \times \mathbb{Z}_2 \times \mathbb{Z}_3
\]

can achieve consistency with LHC, neutrino data

(26-parameter fit)

String Models:

variety of possibilities, not necessarily just minimal Type I seesaw

\[\nu_R\] candidates often not pure gauge singlets

explorations of Type I seesaw in heterotic orbifolds \cite{Giedt et al.; Buchmuller et al.}

braneworlds: exponentially suppressed Yukawas 

\cite{see e.g. Langacker for reviews}

“Mixed” scenarios with seesaw and R-parity violation

e.g. G2 models \cite{Acharya et al. ’16;...}
Concluding Remarks

The SM flavor puzzle is a difficult, intriguing problem — we’re just beginning to scratch its surface!

Most important question: **Majorana** or **Dirac** neutrinos?

New insights/approaches from the lepton data

Naturalness/testability tradeoff

Lots of ideas, lots of room for more

Stay tuned!