The Influence of DE on the Expansion Rate of the Universe and its Effects on DM Relic Abundance

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Based on:

Standard Cosmology
ΛCDM model

- Successful explaining physical phenomena since Big Bang Nucleosynthesis (BBN) epoch at MeV temperatures:
  - Primordial abundances of light elements, Cosmic Microwave Background, etc.
- Expansion history since freeze-out of DM and the precise measurement of the DM relic abundance (27%) fix the annihilation cross-section $\langle \sigma v \rangle$ for a given DM mass.
- The physics describing the Universe’s evolution from the end of inflation (reheating) to just before BBN remains relatively unconstrained.
Thermal Cold Dark Matter

- DM composed of weakly interacting particles.
- In thermal equilibrium during radiation dominated era.
- DM becomes non-relativistic and then decouples before BBN.
  - Abundance \( Y \equiv \frac{n}{s} \) freezes out.
- The Boltzmann equation describes the evolution of the DM abundance.

\[
\frac{x}{Y} \frac{dY}{dx} = -\frac{\Gamma}{H_{GR}} \left( 1 - \left( \frac{Y_{eq}}{Y} \right)^2 \right).
\]

where \( \Gamma \equiv n\langle \sigma v \rangle = Y_s\langle \sigma v \rangle \) and \( x = m/T \).
Evolution is governed by the factor $\frac{\Gamma}{H_{GR}}$.

- $\Gamma \gtrsim H_{GR} \Rightarrow Y = Y_{eq} \rightarrow$ thermal equilibrium.
- $\Gamma \approx H_{GR} \rightarrow$ freeze out.
- $\Gamma \lesssim H_{GR} \Rightarrow Y = Y_{\infty} \rightarrow$ decoupled evolution.
Thermal Evolution of Abundance

Increasing $\langle \sigma v \rangle$

DM Content

$$\Omega_{DM} = \frac{\rho_0}{\rho_c} = \frac{m s_0 Y_\infty}{\rho_c} \approx 0.27$$

$$\langle \sigma v \rangle \approx 2.1 \times 10^{-26} \text{cm}^3/s$$
On the other hand...

Fermi-LAT\(^1\) and Planck\(^2\) experiments have been exploring upper bounds.

\(^1\)arXiv: 1611.03184
\(^2\)arXiv: 1502.015889
Predicting different annihilation cross-section

- Modified expansion rate prior to BBN, due to a modified gravity.
- Scalar-Tensor Theories are good candidates before big-bang nucleosynthesis\(^3\).
  - Gravitational interactions are mediated by a scalar field and a tensor (the metric).
  - Ivonne Zavala's talk, "String-DM/DE"
- Larger or smaller \(\langle \sigma v \rangle\) for a given DM mass

\(^3\)arXiv:0403614, 0912.4421,1508.05174
The metrics

Two frames of references connected by

$$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu \phi \partial_\nu \phi$$

where $C(\phi)$ is the conformal coupling and $D(\phi)$ is the disformal coupling.

Jordan Frame, $\tilde{g}_{\mu\nu}$

- Scalar field couples through the gravitational sector.
- $S_{Mat}(\tilde{g}_{\mu\nu}, \Psi_m)$. By construction, observables such as mass, length and time take their standard interpretation.

$$\frac{\tilde{x} \, d\tilde{Y}}{\tilde{Y} \, d\tilde{x}} = -\frac{\tilde{\Gamma}}{\tilde{H}} \left( 1 - \left( \frac{\tilde{Y}_{eq}}{\tilde{Y}} \right)^2 \right)$$
Scalar-Tensor Theories of Gravity
Frames of reference

**Einstein Frame, $g_{\mu\nu}$**

- Scalar field couples through the matter sector, $S_{\text{Mat}} (g_{\mu\nu}, C(\phi), D(\phi)\partial\mu\phi\partial\nu\phi, \Psi_m)$
- Physical quantities measured in this frame are spacetime dependent.
- Advantage is that the Einstein equations take the usual form, $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 \left( T^\phi_{\mu\nu} + T_{\mu\nu} \right)$
- There is also an equation describing the evolution of the scalar field.
- Solve equations in this frame, to obtain the expansion rate $H$ and $\phi$ evolution. Then, go back to Jordan frame to find $\tilde{H}$.

In this talk, I only focus on the conformal scenario ($D(\phi) = 0$). Thus $\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu}$
Equation of State

It is a frame invariant quantity:

$$\omega = \frac{p}{\rho} = \frac{\tilde{p}}{\tilde{\rho}} = \tilde{\omega}$$

During the radiation dominated era,

$$1 - 3\tilde{\omega} = \frac{\tilde{\rho} - 3\tilde{p}}{\tilde{\rho}} = \sum_A \frac{\tilde{\rho}_A - 3\tilde{p}_A}{\tilde{\rho}}$$

where $\tilde{\rho}_A$ and $\tilde{p}_A$ are the contribution of SM particles and $\tilde{\rho} \sim \pi^2 g_{\text{eff}}(\tilde{T})\tilde{T}^4/3$. 

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Equation of State

\[ \tilde{p} = \tilde{\omega} \tilde{\rho} \]

- Radiation domination, \( \tilde{\omega} \approx \frac{1}{3} \).
- Matter domination, \( \tilde{\omega} = 0 \).
- Dark Energy domination, \( \tilde{\omega} = -1 \).
Conformal Scenario

Evolution of scalar field is dictated by,

$$\frac{2}{3B} \varphi'' + (1 - \omega) \varphi' + 2(1 - 3\omega)\alpha(\varphi) = 0.$$  

where $N = \ln a/a_0$, $B = 1 - \varphi'^2/6$, $\alpha(\varphi) = \frac{d\ln C^{1/2}}{d\varphi}$, $\omega = \frac{p}{\rho}$. 
In the plot below $C(\varphi) = (1 + 0.1 e^{-8 \varphi})^2$ and $(\varphi_0, \varphi'_0) = (0.2, -0.99)$
Conformal Scenario
Comparing $\tilde{\Gamma}$ and $\tilde{H}$
The Boltzmann equation becomes

\[
\frac{\tilde{x}}{\tilde{Y}} \frac{d\tilde{Y}}{dx} = -\frac{\Gamma}{\tilde{H}} \left(1 - \left(\frac{\tilde{Y}_{eq}}{\tilde{Y}}\right)^2\right).
\]

Conformal Scenario
Relic Abundance for 130 GeV
Conformal Scenario

Dark Matter Annihilation Cross Section

\[ \langle \sigma v \rangle / 10^{-26} \text{ (cm}^3 \text{s}^{-1}) \]

Standard

Conformal

\[ \langle \sigma v \rangle \]

\[ m(\text{GeV}) \]

arXiv: 1612.05553
Conclusions

- ST present an attractor mechanism to standard cosmology prior to BBN.
- DM annihilation cross-section larger and smaller than the one predicted by standard cosmology, consistent with the experimental bounds.
- Significant variation in the evolution of abundance for high masses (Presented example for 1000 GeV)
Scalar-Tensor Theories of Gravity

Frames of reference

Two frames of references connected by

$$
\tilde{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi) \partial_\mu \phi \partial_\nu \phi
$$

**Jordan Frame, \( \tilde{g}_{\mu\nu} \)**

- Scalar field couples through the gravitational sector.
- As an example, the action for \( D = 0 \) is given by,

$$
S_{tot} = \frac{1}{2\kappa_{GR}^2} \int d^4x \, \sqrt{-\tilde{g}} \left[ \frac{1}{C} \tilde{R} - \tilde{g}^{\mu\nu} Z(\phi) \partial_\mu \phi \partial_\nu \phi - 2U(\phi) \right] + S_{Mat}(\tilde{g}_{\mu\nu}, \Psi_m).
$$

By construction, observables such as mass, length and time take their standard interpretation.

$$
\frac{\tilde{x}}{\tilde{Y}_{eq}} \frac{d\tilde{Y}}{dx} = -\tilde{H} \left( \left( \frac{\tilde{Y}}{\tilde{Y}_{eq}} \right)^2 - 1 \right)
$$
Einstein Frame, $g_{\mu\nu}$

- Scalar field couples through the matter sector.

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R - \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial \phi)^2 + V(\phi) \right]$$

$$+ S_{\text{Mat}} (C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu \phi \partial_\nu \phi, \Psi_m).$$

- Physical quantities measured in this frame are spacetime dependent.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 \left( T^\phi_{\mu\nu} + T_{\mu\nu} \right)$$
Scalar-Tensor Theories of Gravity
Cosmological equations

FRW metric $g_{\mu\nu}$,

$$ds^2 = -dt^2 + a(t)^2 dx_i dx^i.$$

Einstein and scalar field equations become

$$H^2 = \frac{\kappa^2}{3} \left[ \rho_\phi + \rho \right],$$

$$\dot{H} + H^2 = -\frac{\kappa^2}{6} \left[ \rho_\phi + 3p_\phi + \rho + 3p \right],$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + Q_0 = 0.$$

where $H = \frac{\dot{a}}{a}$. 
With the solution for the scalar field we find $\tilde{H}$ as follows,

$$
\tilde{H} = \frac{C^{1/2}(\varphi)}{C^{1/2}(\varphi_0)} \frac{1}{(1 - \alpha(\varphi)\varphi') \sqrt{1 - \frac{(\varphi')^2}{6(1-\alpha(\varphi)\varphi')^2}}} \frac{1}{\sqrt{1 + \alpha^2(\varphi_0)}} H_{GR}
$$

where $'$ denotes derivative w.r.t $\tilde{N} = \ln \tilde{a}/\tilde{a}_0$. 

Solar system tests of gravity impose constraints\textsuperscript{5}.

- $\alpha_0^2 \lesssim 10^{-5}$
- $\alpha_0' = \left. d\alpha / d\varphi \right|_{\varphi_0} \gtrsim -4.5$.
- $\frac{\tilde{H}}{H_{GR}}$ order 1 before the onset of BBN.

\textsuperscript{5} arXiv: 0009034. 0103036