

TxCETP Course Component

Coordinate, Transformational, and Vector Geometry

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I. Introduction

Objectives: The teacher understands:

- Measurement as a process.
- Geometries, in particular Euclidean geometry, as axiomatic systems.
- The results, uses, and applications of Euclidean geometry.
- Coordinate, transformational, and vector geometry and their connections.

Prerequisites: Algebra I, Geometry, Algebra II, Pre-Calculus

Materials: Graph paper
Calculator
Straight edge

Game Plan: This is intended primarily for use in capstone courses for pre-service mathematics teachers.

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II. Activities

1. Quadrilateral $ABCD$ has vertices $(-2, -3)$, $(-1, 1)$, $(3, -5)$, and $(4, -1)$. The figure is to be translated so that the image of point C is the origin.
 - a. Use graph paper to sketch quadrilateral $ABCD$ and its image.
 - b. Does the image have the same or opposite orientation as the original figure?
 - c. Are the two figures congruent?
 - d. Find the vector that describes this translation.
 - e. Find an equation that uses matrix addition to describe this translation.
 - f. What special quadrilateral is $ABCD$? Use coordinate geometry to prove this.
 - g. Find the area of quadrilateral $ABCD$.
2. Quadrilateral $EFGH$ has vertices $(-4, 1)$, $(0, 6)$, $(3, 1)$, and $(3, 6)$. The figure is to be reflected over the x -axis.
 - a. Use graph paper to sketch quadrilateral $EFGH$ and its image.
 - b. Does the image have the same or opposite orientation as the original figure?
 - c. Are the two figures congruent?
 - d. Find an equation that uses matrix multiplication to describe this reflection.
3. Quadrilateral $EFGH$ has vertices $(-4, 1)$, $(0, 6)$, $(3, 1)$, and $(3, 6)$. The figure is to be reflected over the y -axis.
 - a. Use graph paper to sketch quadrilateral $EFGH$ and its image.
 - b. Does the image have the same or opposite orientation as the original figure?
 - c. Are the two figures congruent?
 - d. Find an equation that uses matrix multiplication to describe this reflection.
4. Quadrilateral $EFGH$ has vertices $(-4, 1)$, $(0, 6)$, $(3, 1)$, and $(3, 6)$. The figure is to be reflected over the line $y = x$.
 - a. Use graph paper to sketch quadrilateral $EFGH$ and its image.
 - b. Does the image have the same or opposite orientation as the original figure?
 - c. Are the two figures congruent?
 - d. Find an equation that uses matrix multiplication to describe this reflection.
5. Triangle ABC has vertices $(-2, 5)$, $(1, 3)$ and $(4, -1)$.
 - a. Use graph paper to sketch triangle ABC and its image under a rotation of 30 degrees about the origin.
 - b. On another grid, sketch triangle ABC , its image under a rotation of 90 degrees, 180 degrees, and 270 degrees about the origin. Find an equation that uses matrix multiplication to describe each of these rotations.
 - c. Find the area of triangle ABC .

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6. Triangle ABC has vertices $(3, 1)$, $(1, 4)$, and $(6, 3)$.
 - a. Find the coordinates of a dilation centered at $(0, 0)$ with scale factor 1.5. Use graph paper to sketch triangle ABC and its image under this transformation.
 - b. Find an equation that uses matrix multiplication to describe this transformation.
 - c. Find the area of triangle ABC and find the area of its image.

7. Triangle ABC has vertices $(3, 1)$, $(1, 4)$, and $(6, 3)$.
 - a. Write the equations of the altitudes of triangle ABC .
 - b. Find the point at which the altitudes are concurrent.
 - c. Write the equations of the medians of triangle ABC .
 - d. Find the point at which the medians are concurrent.
 - e. Use graph paper to sketch triangle ABC along with its image under a reflection over the line $x = 2$ followed by a reflection over the line $x = 5$. Explain how the image is related to the original triangle.
 - f. Use graph paper to sketch triangle ABC along with its image under a reflection over the line $x = 2$ followed by a reflection over the line $y = 5$. Explain how the image is related to the original triangle.

8. Determine whether each of the following quadrilaterals is a trapezoid, a parallelogram, rectangle, or a square. Use coordinate geometry to prove your statement.
 - a. $A(1, 0)$, $B(2, -2)$, $C(4, -1)$, $D(7, 3)$
 - b. $A(1, 5)$, $B(-3, 5)$, $C(-3, -1)$, $D(1, -1)$

9. Use coordinate geometry to prove the following:
 - a. If a segment joins the midpoint of two sides of a triangle, then it is parallel to the third side and half as long.
 - b. The locus of all points equidistant from two given points is the perpendicular bisector of the line segment joining the two points.
 - c. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.
 - d. The midpoint of the hypotenuse of a right triangle is equidistant from the triangle's vertices.
 - e. The diagonals of a rectangle are equal in length.
 - f. Lines segments joining the successive midpoints of the sides of any quadrilateral form a parallelogram.
 - g. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
 - h. If two coplanar lines are perpendicular to the same line, then the two lines are parallel.

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10. A set of points is said to be symmetric with respect to a line k if the reflection image of any point of the figure about line k is also a point of the figure. A figure has rotational symmetry if there is some point P such that the figure can be rotated and made to coincide with itself. Point symmetry occurs when a rotation of 180 degrees maps a figure onto itself. Which letters of the alphabet have line symmetry? Rotational symmetry? Point symmetry?
11. A tessellation is a repeating pattern that completely covers a plane region with no holes or gaps. Explain why every quadrilateral and every triangle tessellates.
12. A circle is a set of points in a plane equally distanced from a fixed point called the center. Let the center be (h, k) and the distance from the center to the circle be r .
 - a. Use coordinate geometry and the definition of the circle to derive an equation for this circle.
 - b. A hiker is lost in the woods. He was last spotted at coordinates $(20, 70)$ on the search team's map. Since he was last seen, the hiker had time to travel at most 12 grid units. Find an equation that defines the perimeter of the search area.
13. A parabola is the set of all points in a plane equidistant from a particular line called the directrix and a particular point called the focus. Let the directrix be $y = -c$ and the focus be located at $(0, c)$.
 - a. Use coordinate geometry and the definition of the parabola to derive an equation for this parabola.
 - b. Suppose this parabola undergoes a translation of $\langle h, k \rangle$. Write the equation of its image.
 - c. The towers of a suspension bridge are 600 feet apart and 150 feet above the road. The cable between the towers has the shape of a parabola. Midway between the towers the cable is 10 feet above the road. What is the height of the cable 50 feet from the tower?
 - d. A satellite dish is a paraboloid of revolution. Its diameter is 6 feet and it is 3 feet deep. How far from the vertex of the paraboloid should the receiving antenna be placed?
14. An ellipse is the set of all points in a plane whose distances from two fixed points in the plane have a constant sum. Let the ends of the major axis be $(-a, 0)$ and $(a, 0)$ and let the two fixed points be $(-c, 0)$ and $(c, 0)$ where $c < a$.
 - a. Use coordinate geometry and the definition of the ellipse to derive an equation for this ellipse.
 - b. Suppose this ellipse undergoes translation $\langle h, k \rangle$. What is the equation of its image?
 - c. A semielliptical archway over a two-lane (one lane in each direction) busy highway is ten feet high and 30 feet wide. You are driving a truck that is 8 feet wide and 9 feet high. It is urgent that you reach your destination without having an accident as you are hauling the TExES exams to the testing center for all the mathematics students waiting in excited anticipation of taking the test. Are you likely to succeed in this ever so important mission? Justify your answer numerically.

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- d. A lithotripter is used to disintegrate kidney stones. The ellipse used to generate the ellipsoid of the lithotripter has a major axis of 12 feet and a minor axis of 5 feet. Where should the shock wave source and the patient be placed to obtain the best results?
15. A hyperbola is the set of all points in a plane whose distances from two fixed points (called the foci) in the plane have a common difference. Let the ends of the focal axis be $(-a, 0)$ and $(a, 0)$ and let the two fixed points be $(-c, 0)$ and $(c, 0)$ where $c > a$.
- a. Use coordinate geometry and the definition of the hyperbola to derive an equation for this hyperbola.
- b. Suppose this hyperbola undergoes translation $\langle h, k \rangle$. What is the equation of its image?