

III. Transformations

A general transformation $M: (x, y) \rightarrow (ax, by)$ stretches the plane by factors that are not necessarily the same in the x and y directions. Map triangle ABC to triangle $A'B'C'$ using the transformation $M: (x, y) \rightarrow (x, 2y)$. On the same grid, show triangle ABC and triangle $A'B'C'$. The diagram we have just drawn represents a roof along with its support posts. The eave is segment OC' where O is the origin. Support posts are placed every 2 feet on the horizontal beam starting at the eave and running parallel to the eave.

A. Find the points D' through T' by using the above transformation on points D through T .

B. Calculate the slope of each side of the roof. Show the points that you used and the calculations you performed.

C. Explain the significance of the slopes being numerically the same, but opposite in signs.

D. Calculate the measure of angles $C'A'B'$ and $C'B'A'$ using one of your trigonometric formulas. Show your calculations.

E. Explain why these two angles must be congruent within the realm of this application.

F. Explain how the slope of a line is related to the particular trigonometric function you used in Part C.

G. The peak of the roof is located at which point on our diagram?

H. Write the equation of lines $A'C'$ and $B'C'$ by using the slope intercept form.

I. Are these lines perpendicular? How can you tell? Can you make a conjecture about under what conditions they will be perpendicular?

J. Find the areas of the following triangles: $A'D'E'$, $A'F'G'$, $A'H'I'$, $A'J'K'$, and $A'OC'$.

K. Set up a sequence containing the ratios of the area of each triangle to the area of the next triangle.

L. Look for any patterns you can find in this sequence. Write a formula to represent the n th term of this sequence.

M. Find the lengths of the following legs of these triangles: $A'D'$, $A'F'$, $A'H'$, $A'J'$, and $A'O$.

N. Set up a sequence containing the following ratios: $A'D'/A'F'$, $A'F'/A'H'$, $A'H'/A'J'$, and $A'J'/A'O$.

O. Look for any patterns you can find in this sequence. Write a formula to represent the n th term of this sequence.

P. Find the lengths of the following legs of these triangles: $D'E'$, $F'G'$, $H'I'$, $J'K'$, and OC' .

- Q. Set up a sequence containing the following ratios: $D'E'/F'G'$, $F'G'/H'I'$, $H'I'/J'K'$, $J'K'/O'C'$.
- R. Look for any patterns you can find in this sequence. Write a formula to represent the n th term of this sequence.
- S. Explain how the formulas from parts L', O', and R' are related.
- T. List the x coordinates of the points A', E', G', I', K', and C'. These coordinates form an arithmetic sequence of numbers. What is the common difference?
- U. List the y coordinates of the points A', E', G', I', K', and C'. These coordinates form an arithmetic sequence of numbers. What is the common difference?

V. What is the ratio of the common difference of the y coordinates to the common difference of the x coordinates?

W. What can you say about the relationship between the slope of a line \mathcal{L} and the slope of the transformation of line \mathcal{L} under the mapping $\mathcal{L}: (x,y) \rightarrow (ax, by)$