

## **TxCETP Course Component**

# **Mathematical Processes and Perspectives**

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### Table of Contents

- I. Introduction
- II. Activities

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# I. Introduction

**Objectives:** The beginning teacher will:

- Understand mathematical reasoning and problem solving
- Understand mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts

**Prerequisites:** Algebra I, Geometry, Algebra II, Pre-Calculus

**Materials:** High School Mathematics Text Books

Internet Access

Microsoft Word

Graphing Calculator

CBR

Graph paper

**Game Plan:** This is intended primarily for use in capstone courses for pre-service mathematics teachers.

**Resources:**

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## II. Activities

- I. Define inductive reasoning and give an example that middle school students or high school students would find interesting.
- II. Define deductive reasoning and give an example that middle school or high school students would find interesting.
- III. Explain indirect reasoning and give an example that would be of interest to a teenager.
- IV. Tell whether the reasoning process in each of the following statements is inductive, deductive, or indirect.
  - a. If  $4x - 15 = 0$ , then  $4x = 15$  and  $x = \frac{15}{4}$ .
  - b. A student notices that Mrs. M has not given homework on any of the first four Wednesdays of the semester. The student concludes that Mrs. M never gives homework on Wednesdays.
  - c. Given the pattern 1, 4, 2, 8, 4, 16, 8, \_\_\_\_, you predict the next term is 32.
  - d. You are ill. The physician diagnoses your illness.
  - e. Ky enters his house carrying a dry umbrella. You conclude that it is not raining because if it were raining, the umbrella would be wet.
- V. Determine if each statement given below is true or false. If false, give a counterexample. You may use words or pictures for your counterexample.
  - a. If a triangle is isosceles, then it can not be obtuse.
  - b. All odd integers less than ten are prime.
  - c. If a line lies in a vertical plane, then the line is a vertical line.
- VI. Explain the Principle of Mathematical Induction.
- VII. Discuss the Tower of Hanoi Problem. Find an interactive website where you can play the game on your computer. Cite the URL for this website.
- VIII. Using mathematical induction, prove that the minimum number of moves required to move a stack of  $n$  washers in a Tower of Hanoi game is  $2^n - 1$ .
- IX. Explain the steps required to write an indirect proof.
- X. Prove that the diagonals of a trapezoid do not bisect each other. Use a clearly thought out and justified indirect proof.
- XI. Draw a regular pentagon  $ABCDE$ . Let  $F$  be the intersection of segment  $AC$  and segment  $BD$ . What special kind of quadrilateral is  $AFDE$ ? Write a deductive proof in paragraph form to justify your answer.

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- XII. George Polya is sometimes called the father of modern problem solving. Explain in detail his four steps to problem solving. See the following websites for help: [www.math.grin.edu/~rebelsky/ProblemSolving/Essays/polya.html](http://www.math.grin.edu/~rebelsky/ProblemSolving/Essays/polya.html) and [www.lhup.edu/~agrissin/polya.htm](http://www.lhup.edu/~agrissin/polya.htm)
- XIII. Go to the first website listed in XII. On the bottom, click on problems. Click on Topical Catalogue. Now, choose one of the following problems from their list:
- |                            |                   |
|----------------------------|-------------------|
| Worlds in Collision        | Pick Your Grade   |
| Rendezvous at Noon         | Leap Birthdays    |
| Up and Down                | Zoomsters         |
| Triangles on a Chess Board | Changing the Base |
| Break It Up                |                   |
- Find a solution to the problem you chose. Explain the steps in solving this problem and explain your reasoning thoroughly.
- XIV. Suppose you are standing 15 feet away from two children in the class who begin fighting. You walk at a constant rate of 2 feet per second toward the children.
- Draw a pictorial representation of this situation.
  - Make a table of values showing the relationship between  $t$ , the time spent walking toward the children, and  $d$ , your distance away from the children. Use one second increments for  $t$ .
  - Find an equation to model this situation. State any restrictions on the value of  $t$ .
  - Graph the equation you found in part c.
  - Using your equation, find out how long it will take for you to reach the children.
  - Using your graphing calculator and your graph, find out how long it will take to reach the children. Explain the procedure.
- XV. Set a CBR on a table. Start out on the opposite side of the room and walk at a constant rate toward the CBR for 4 seconds. Now, stand still for 2 seconds. Then, walk at a constant rate for 3 seconds.
- Run the ranger program to obtain a graph of distance verses time.
  - Model this situation algebraically.
- XVI. A Ferris wheel has a diameter of 50 feet and makes one revolution every 45 seconds. The center of the wheel is 28 feet above the ground.
- Draw a pictorial representation for this situation.
  - Make a table of values (at least 10 points) showing the relationship between  $t$ , the time after reaching the low point, and  $h$ , the height above the ground.
  - Find an equation to model this situation.
  - Graph your equation.
  - Use your graph and graphing calculator to find the time at which the rider is 40 feet above the ground.
  - Use your model to algebraically find the time at which the rider is 40 feet above the ground.